Leaving Cert Mathematics Grinds - Week 3

Topic: Algebra III

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Leaving Cert
Mathematics
Grinds

Week 3:
Algebra III

Sound & Visual Check

“I am now talking....”

“If you **<u>cannot</u>** hear me or see my screen please say “Cannot hear/see you” on the chat.

“If some of you can’t hear me, please restart your computer and join the class again.”

Leaving Cert Mathematics Grinds

Week 3:
Algebra III

Lesson Overview:

By the end of this lesson you should:

- **Understand** how to solve simultaneous equations
- **Understand** how to solve logarithmic and exponential formulae.
- **Understand** how to determine the roots of a cubic equation.
- **Know** how to use the logarithmic and power rules in the formula tables.
- **Know** how to manipulate surds.
- **Have** a better understanding of logarithmic functions and their uses.

Solve the inequality

$$\frac{x+4}{4} < \frac{x-1}{7} + \frac{1}{5}$$

and show the solution on a number line.

Lets first bring all the x to one side firs

$$\frac{x+4}{4} - \frac{x-1}{7} < \frac{1}{5}$$

Change the left hand side to one fraction

$$\frac{7(x+4) - 4(x-1)}{4(7)} < \frac{1}{5}$$

$$\frac{7x + 28 - 4x + 4}{28} < \frac{1}{5}$$

$$\frac{3x + 32}{28} < \frac{1}{5}$$

$$3x + 32 < \frac{1}{5}(28)$$

$$3x < \frac{28}{5} - 32$$

$$3x < \frac{-132}{5}$$

$$\boxed{x < \frac{-132}{15}}$$

Graph the solution of the set A of

$$-2 \leq \frac{3x+1}{4}$$

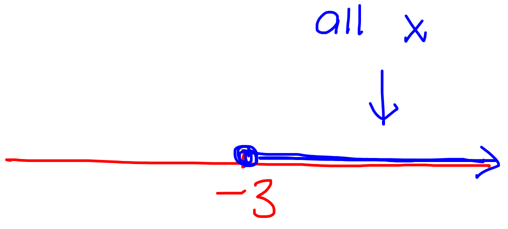
Graph the solution of the set B of

$$8 < \frac{4x-4}{2}$$

What is the intersection of A and B?

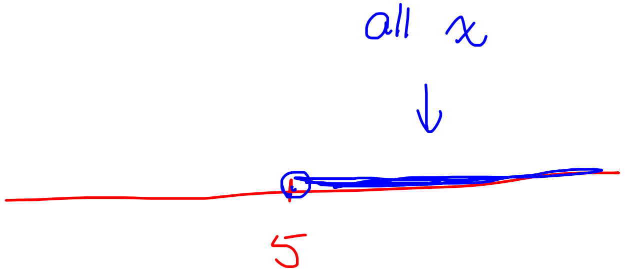
1

$$-8 \leq 3x + 1$$
$$-9 \leq 3x$$
$$-3 \leq x$$



2

$$16 < 4x - 4$$
$$20 < 4x$$
$$5 < x$$



Where do they intersect?

↳ This means what are the range of numbers that are in both A and B?

All $x < 5$! → These are in both A and B!

Find all the real values for x for which $2x^2 + x - 15 > 0$

We can factorise this using quadratics!

Just pretend this is an equal sign until the end

$$2x^2 + x - 15 > 0$$

$$(2x - 5)(x + 3) > 0$$

So ether

$$2x - 5 > 0 \qquad\qquad x + 3 > 0$$

$$2x > 5 \qquad\qquad x > -3$$

$$x > \frac{5}{2}$$

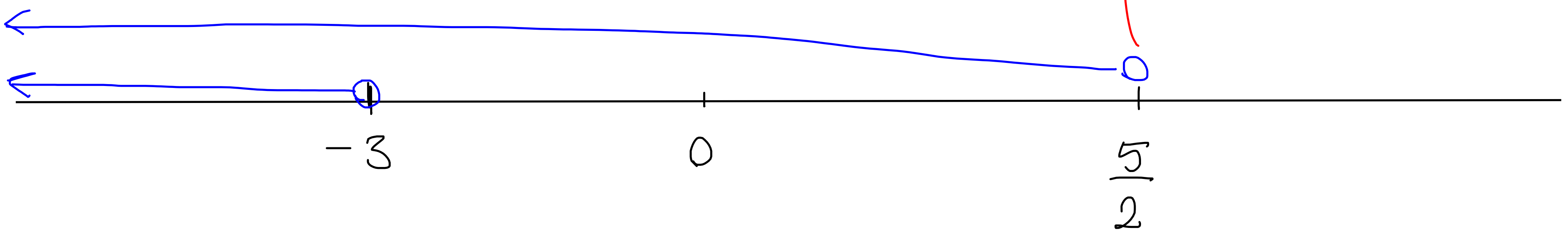
So ~~at~~ $x > \frac{5}{2}$ and $x > -3$!

Question 2 **(25 marks)**

(a) Find the set of all real values of x for which $2x^2 + x - 15 \geq 0$.

To draw the question we did previously

empty circle means its not including!



So the range of values are

$$x < \frac{5}{2} \setminus \{-3\}$$

This means its not including -3

Find the solution to the inequality

$$x < 3(-x^2 + 2x + 5)$$

Just pretend its an equal sign until the end!

$$x < -3x^2 + 6x + 15$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$3x^2 - 6x - 15 + x < 0$$

$$3x^2 - 5x - 15 < 0$$

Solve this quadratic using the -b formula!

$x < 3.22$ or $x < -1.55$

Discriminent – Used to determine nature of functions roots

- When greater than zero, roots are distinct and real
- When equal to zero, roots are equal and real
- When less than zero roots are complex

Discrimenent!

$b^2 - 4ac$

- What does this mean on a graph?

Determine the nature of the roots of the following

i) $x^2 - 6x + 4 = 0$ $\quad a = 1 \quad b = -6 \quad c = 4$

ii) $x^2 - 6x + 9 = 0$ $\quad a = 1 \quad b = -6 \quad c = 9$

iii) $4x^2 + 8x + 20 = 0$ $\quad a = 4 \quad b = 8 \quad c = 20$

i) $b^2 - 4ac \longrightarrow (-6)^2 - 4(4)(1) = 36 - 16 = 20 > 0$

$\Rightarrow$ Real distinct roots

ii) $b^2 - 4ac \longrightarrow (-6)^2 - 4(1)(9) = 36 - 48 = -12 < 0$

$\Rightarrow$ Imaginary roots

iii) $b^2 - 4ac \longrightarrow 8^2 - 4(4)(20) = 64 - 320 = -256 < 0$

$\Rightarrow$ Imaginary roots.

$$\frac{3x-1}{x+1} < 1 \qquad \times \quad (x+1)^2$$

$$(x+1)^{\cancel{2}}\left(\frac{3x-1}{\cancel{x+1}}\right) < (x+1)^2$$

$$(x+1)(3x-1) < (x+1)^2$$

$$3x^2+2x-1 < x^2+2x+1$$

$$2x^2-2<0$$

$$2x(x-1)<0$$

$$2x<0 \qquad\qquad x-1<0$$

$$x<0 \qquad\qquad x<+1 \quad !$$

$$|2x+1| = 5$$

This is called the modulus or the absolute value!

Square both sides to get rid of the modulus

$$(2x+1)^2 = 5^2$$

$$4x^2 + 4x + 1 = 25$$

$$4x^2 + 4x - 24 = 0$$

$$x^2 + x - 6 = 0$$

$$(x+3)(x-2) = 0$$

So either $x = -3$ or $x = 2$!

Again we square both sides!

$$2|x+1| = |x-2|$$

$$[2|x+1|]^2 = [|x-2|]^2$$

$$2^2(x+1)^2 = (x-2)^2$$

$$4(x^2+2x+1) = x^2-4x+4$$

$$4x^2+8x+4 = x^2-4x+4$$

$$3x^2+12x = 0$$

$$3x(x+4) = 0$$

So either $3x = 0$ or $x = -4$!

$x = 0$

Question 2 **(25 marks)**

(a) Find the range of values of x for which $|x - 4| \geq 2$, where $x \in \mathbb{R}$.

Here we square both sides

$$(x-4)^2 \geq 2$$

$$x^2 - 8x + 16 \geq 2$$

$$x^2 - 8x + 14 \geq 0$$

We now use the $-b$ formula to get $x = 4 \pm \sqrt{2}$

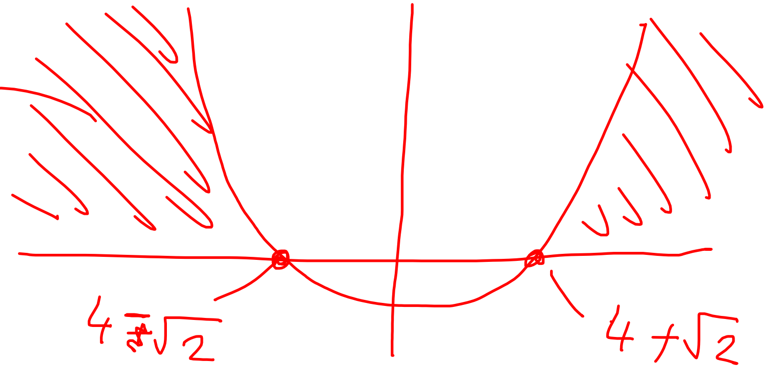
So roots are $x = 4 + \sqrt{2}$ $\quad x = 4 - \sqrt{2}$

So range is

$x < 4 - \sqrt{2}$

$x > 4 + \sqrt{2}$

Shaded area is where ≥ 0!



Next Weeks Lesson:

Leaving Cert
Mathematics
Grinds - Week 4

Topic: Skill Acquisition

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