Junior Cert Mathematics Grinds - Week 2

Topic: Algebraic Expressions & Equations

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Education is the key to success

Junior Cert Mathematics Grinds

Week 2: Algebraic Expressions & Equations

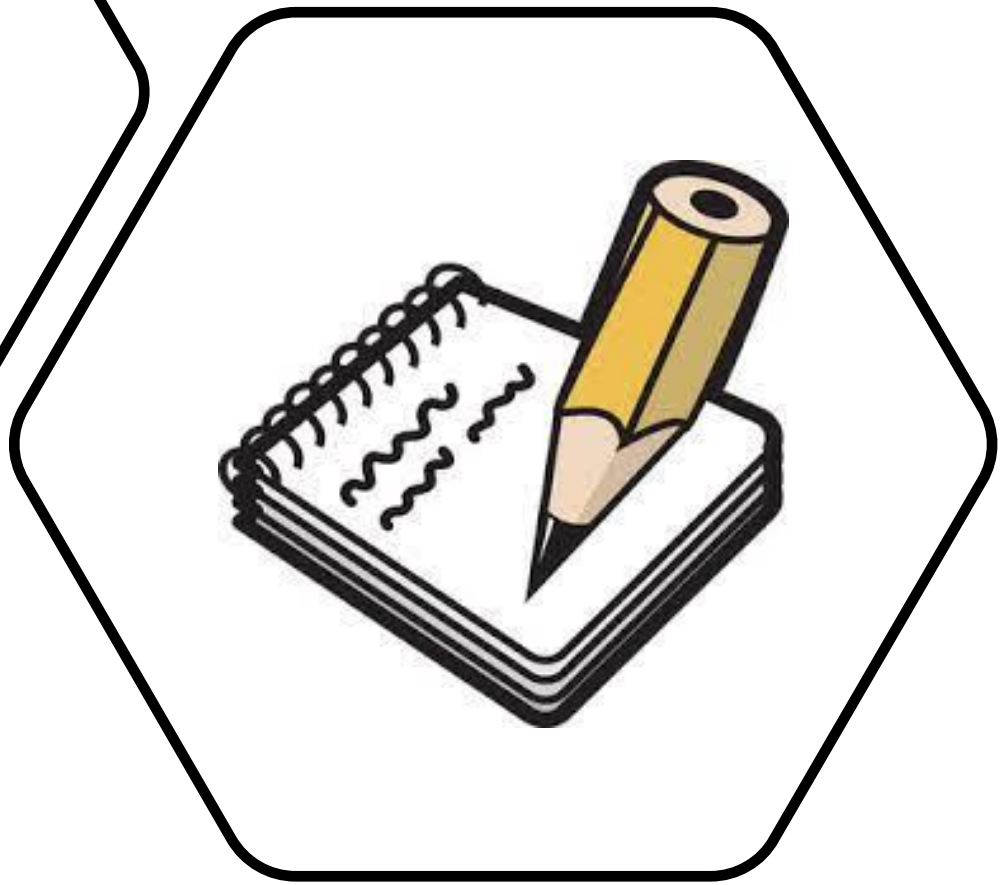
Sound & Visual Check

“I am now talking….”

“If you **cannot** hear me or see my screen please say “Cannot hear/see you” on the chat.

“If some of you can’t hear me, please restart your computer and join the class again.”

Make sure you have a pen and paper to hand



Junior Cert Mathematics Grinds

Week 2: Algebraic Expressions & Equations

Lesson Overview:

By the end of this lesson you should:

- **Understand** how to add, subtract, multiply and divide algebraic expressions.
- **Understand** how to solve linear equations with one variable.
- **Know** how to identify linear equations
- **Know** how to solve simultaneous linear equations with two unknown variables.
- **Have** the tools and knowledge to solve a wide range of problems involving linear equations.

Why do we need Algebra!?

- The use of Algebra in mathematics has evolved over centuries. It can be traced back to Ancient Babylonian and Egyptian times.
- Algebra enables us to solve a vast array of problems involving numbers and unknown variables. It is an essential skill for continuing in the field of Science, Technology, Engineering or Mathematics.

Why do we need algebra!?

- However, whatever career path you choose, you'll find that a good understanding of algebra will help you make many decisions regarding personal finances, including mortgage choice, bank savings accounts and phone plan choices.
- Being able to solve the types of equations we are looking at today will help you in the future make wise and informed choices regarding such matters.

Let's look at a real-life example

- You need to get a new phone. The cheapest you have seen the phone sim-free is an upfront cost of €250. You have seen a sim-only phone plan for €20 per month that meets all your needs. Alternatively, you can get the phone on contract for €100 if you sign up to 24 months monthly line rental. What's the maximum monthly contract payment you would want to sign up to as not to spend more money over the 2 year period?

$$250 + (24 \times 20) = 100 + 24x$$
$$250 + 480 - 100 = 24x$$
$$\frac{630}{24} = x = €26.25$$

Example 1:

$3(2x^2 + 3x - 1)$

Example 2:

$2x(x + 2)$

Example 3:

$(x + 2)(x - 3)$

Example 4:

$(2x + 1)(3x - 2)$

Example 5:

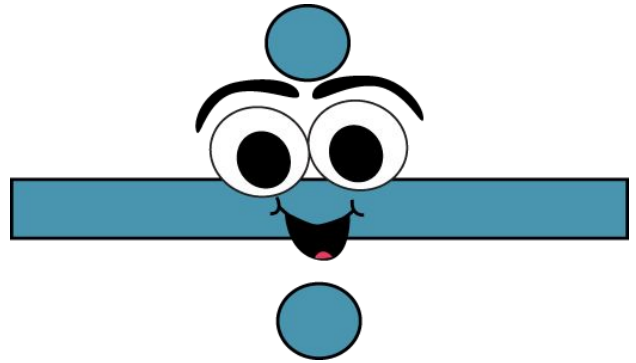
$(2x + 3)^2$

Multiplying Algebraic expressions

- Example 1: $3(2x^2 + 3x - 1) \rightarrow 6x^2 + 9x - 3$
- Example 2: $2x(x + 2) \rightarrow 2x^2 + 4x$
- Example 3: $(x + 2)(x - 3) \rightarrow x^2 - 3x + 2x - 6 \rightarrow x^2 - x - 6$
- Example 4: $(2x + 1)(3x - 2) \rightarrow 6x^2 - 4x + 3x - 2 \rightarrow 6x^2 - x - 2$
- Example 5: $(2x + 3)^2 \rightarrow (2x + 3)(2x + 3) \rightarrow 4x^2 + 6x + 6x + 9$

$\rightarrow 4x^2 + 12x + 9$

Dividing Algebraic Expressions



Example 1: Divide $6x^2 + 11x + 3$ by $3x + 1$

$$
\begin{array}{r|l}
 & \quad 2x \quad +3 \\
\hline
3x+1 & 6x^2+11x+3 \\
 & \underline{\;6x^2+2x\;} \\
 & \quad\;\; 9x+3 \\
 & \quad\;\; \underline{\;9x+3\;} \\
 & \qquad\;\; 0
\end{array}
$$

Example 2: Simplify $\frac{6x^3-26x^2+32}{3x-4}$

$$
\begin{array}{r|l}
 & \quad\;\; 2x^2 \quad -6x \quad -8 \\
\hline
3x-4 & 6x^3-26x^2+0x+32 \\
 & \underline{\;6x^3-8x^2\;} \\
 & \quad\;\; -18x^2+0x \\
 & \quad\;\; \underline{\;-(-18x^2+24x)\;} \\
 & \qquad\;\;\; -24x+32 \\
 & \qquad\;\;\; \underline{\;-(-24x+32)\;} \\
 & \qquad\qquad\;\; 0
\end{array}
$$

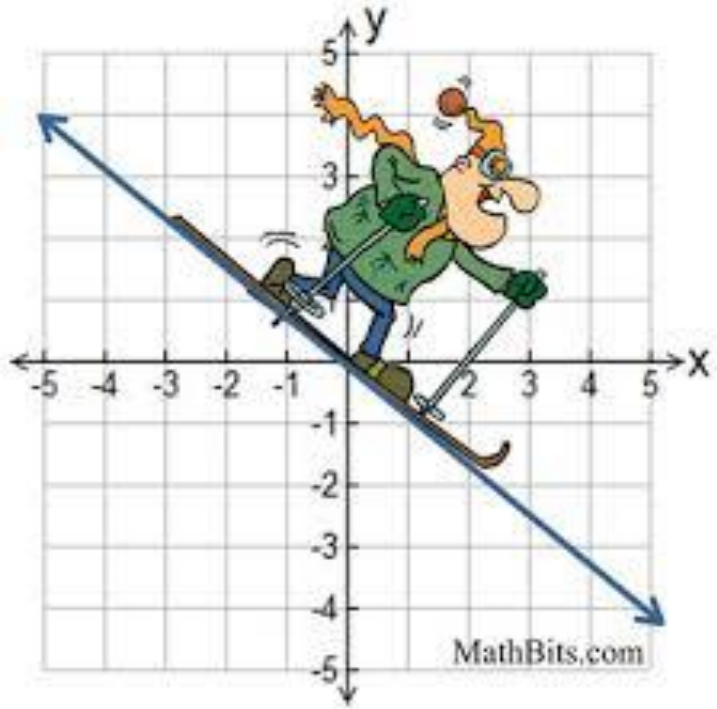
Linear equations

- In **linear equations**, the variables (x, y, a, b etc…) will have a power of 1 (no x^2, x^3, etc…). Linear equations have two variables and they will form a straight line when plotted on a graph.

Here is an example: $2y + 3x = 12$

If we are told x= 2, we can work out y:

$$2y + 3(2) = 12$$
$$2y = 6$$
$$y = 3$$



Simultaneous Equations

- Sometimes we will have two linear equations with the same two variables. We can solve these simultaneous equations to find values of both variables that will be correct for both equations.

$$\begin{array}{rl} 2x + 3y &= 12 \\ x + 3y &= 9 \\ \hline x &= 3 \end{array}$$

Here the y **coefficient** is the same for each equation. We can subtract one equation from the other to find x

$$\begin{aligned} 3 + 3y &= 6 \\ 3y &= 3 \\ y &= 1 \end{aligned}$$

We then substitute x=3 into one of our equations to find y

Simultaneous Equations

- Sometimes the coefficients for x or y will not be the same. In this case we will have to find a common multiple

$$x + 2y = 1$$
$$4x + 3y = 2$$

$$4x + 8y = 4$$
$$4x + 3y = 2$$
$$5y = 2$$
$$y = \frac{2}{5}$$

We can make the x coefficient for both equations 4 by multiplying the first equation by 4

We then substitute $y=\frac{2}{5}$ into one of our equations to find y

$$x + 2(\frac{2}{5}) = 1$$
$$x = 1 - \frac{4}{5} = \frac{1}{5}$$

Question 10 JC OL 2013 P1

Exam Practice!

(a) Find the values of the following expressions if $x = 3$ and $y = 5$.

(i) $5x + 4y$

$5(3) + 4(5)$

$(5 \times 3) + (4 \times 5) = 15 + 20 = 35$

(ii) $x^2 + y^2$

$3^2 + 5^2 = 9 + 25 = 34$

(b) **(i)** Multiply $5(3a - 4b)$.

$(5 \times 3)a + (5 \times -4)b$

$15a - 20b$

(ii) Multiply $x(x - y) + y(x + y)$. Write the answer in its simplest form.

$x^2 - xy + xy + y^2$

$x^2 + y^2$

Exam Practice!

(c) Divide $2x^3 + x^2 - 13x + 6$ by $x + 3$.

$$
\begin{array}{r|l}
x + 3 & 2x^2 - 5x + 2 \\
\hline
 & 2x^3 + x^2 - 13x + 6 \\
 & 2x^3 + 6x^2 \\
 & \quad -5x^2 - 13x \\
 & \quad -5x^2 - 15x \\
 & \qquad\qquad 2x + 6 \\
 & \qquad\qquad 2x + 6 \\
 & \qquad\qquad\quad 0
\end{array}
$$

JC HL 2013 P1

(d) A company employs two drivers, John and David. Each has use of a company car and small van. The company buys €30 worth of Toll Tags for each driver. Each time that a vehicle goes through the M50 Toll, a charge will be deducted from the Toll Tags.

John goes through the M50 toll five times in his car and four times in his small van. He then has €7·90 remaining on his Toll Tags. David goes through the M50 Toll twice in his car and six times in his small van. He then has €8·40 left on his Toll Tags.

Calculate how much it costs for a car and for a small van to go through the M50 Toll.

Exam Practice!

Let's make c cost of Car Toll and v cost of Van toll

$$5c + 4v = 30 - 7.9 = 22.1$$
$$2c + 6v = 30 - 8.4 = 21.6$$

$$v = \frac{22.1 - 5c}{4} = \frac{21.6 - 2c}{6}$$

$$6(22.1 - 5c) = 4(21.6 - 2c)$$
$$132.6 - 30c = 86.4 - 8c$$
$$46.2 = 22c$$
$$2.1 = c = €2.10$$

$$v = \frac{22.1 - 5(2.1)}{4} = 2.9 = €2.90$$

JC HL 2014 P1

Exam Practice!

Question 12 **(Suggested maximum time: 15 minutes)**

(a) Simplify $(6x-3)(2x-1)$.

$12x^2 - 6x - 6x + 3$

$12x^2 - 12x + 3$

(b) Simplify $(3x^3 - 2x^2 - 3x + 2) \div (x-1)$.

$$
\begin{array}{r|l}
 & 3x^2 + x - 2 \\
\hline
x - 1 & 3x^3 - 2x^2 - 3x + 2 \\
 & 3x^3 - 3x^2 \\
 & \quad x^2 - 3x \\
 & \quad x^2 - x \\
 & \quad\quad -2x + 2 \\
 & \quad\quad -2x + 2 \\
 & \quad\quad\quad 0
\end{array}
$$

JC HL 2014 P1

Exam Practice!

(c) (i) Solve the simultaneous equations:

$$2x - 3y = 18$$
$$5x + 9y = -10.$$

Multiply first equation by -3 to make y term coefficient the same in each equation. The subtract this from second equation.

$$\begin{aligned} 5x + 9y &= -10 \\ -6x + 9y &= -54 \\ \hline 11x &= 44 \\ x &= 4 \end{aligned}$$

$$\begin{aligned} 2(4) - 3y &= 18 \\ -3y &= 10 \\ y &= -\frac{10}{3} \end{aligned}$$

Substitute value for x into one of the equations to find y

(ii) Verify your answer to **(c)(i)**.

$$5(4) + 9\left(-\frac{10}{3}\right) = 10$$

Substitute values of x & y into the equation that wasn't used to calculate y in previous part

Quick Plenary Quiz

1) Add these two expressions: 2x = 0

A) x=2, x=3 B) x=2, x=-3 C) x=-2, x=3 D) x=-2, x=-3

2) Factorise: $x^2 + 3x - 10$

$x^2 + 5x - 2x - 10 \rightarrow x(x+5) - 2(x+5) \rightarrow (x+5)(x-2)$

3) Expand $(3x + 2)(2x - 3)$

$6x^2 - 9x + 4x - 6 \rightarrow 6x^2 - 5x - 6$

4) Which of these is an example of a quadratic binomial?

A) $3x^2+2x+1$ B) 5x + 3y C) $6x^2 + 3x$ D) 4x - 5

5) Solve the simultaneous equations:

$3x + 4y = 2$

$2x + 4y = 1$

$x = 1,$ $\quad 2(1) + 4y = 1 \rightarrow 4y = -1 \rightarrow y = -\frac{1}{4}$

Junior Cert Mathematics Grinds

Week 2: Algebraic Expressions & Equations

Lesson Overview:

You should now:

- **Understand** how to multiply and divide algebraic expressions.
- **Understand** how to factorise algebraic expressions.
- **Understand** how to solve linear equations.
- **Know** how to identify linear and quadratic equations.
- **Know** how to solve quadratic equations using factorisation and the quadratic formula.
- **Have** the tools and knowledge to solve a wide range of problems involving linear and quadratic equations.

Next Weeks Lesson:

Junior Cert Mathematics Grinds - Week 3

Topic: Factorising / Solving Quadratic Equations

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